**Report: Present the mathematical problem of fitting the diffusion tensor , which should reference Jiang, et al. [1] and the Background Reading document where appropriate (~1 page).**

**Report: Describe what issues arise due to bad or invalid data at any step of the process, and explain how this is handled, with justification (~1/2 -- 1 page).**

**Introduction**

Diffusion-weighted MRI (magnetic resonance imaging) in the brain allows medical professionals to reconstruct the brain, in order to study brain anatomy and diagnose patients with potential conditions, safely. Patients are exposed to a magnetic field, and the diffusivity of water in different locations of the brain tissue is measured. Thus we are given a single slice of a scan along the axial plane for a patient, with the objective of using this data to estimate the diffusion tensor at each voxel.

From Jiang, et al’s paper ‘ ‘, We have the mathematical equation,

Or 

where *S* is the signal intensity, which decays exponentially as a function of:the constant diffusion tensor  (mm/s),the direction of the diffusion sensitising gradient pulse  (a unit vector in ), and the parameter *b* (s/mm) - the diffusion-weighting factor set by the machine operator . *b* is a scalar that absorbs all the details about the gradient pulse other than its direction, such as its strength and timing and is held constant for all the gradient pulses. For the purposes of our model, we will take b as 1000 s/mm (a typical value); the only variable changing throughout the scan are the directions .

Using the equation above, we can obtain a system of equations, which can be written in matrix form. Since the initial equation is not linear, we will take the logarithms of each side, and construct a linear system of the form , where A is a matrix of , x is our g and b is …

We must take into account noise (corrupt or meaningless samples) and remove unwanted data from our calculations to improve the accuracy. In particular, since we will be taking the logarithm, we require any negative values to be removed from the data set.

Since we cannot find *x* such that , we must find the most fitting solution for x, for which we can use the least squares method. The objective is to minimise the norm of the *residual*: . Solutions to the least squares problem A black background with white text

Description automatically generated can be found by solving the normal equations , finding the QR decomposition  and then solving the *triangular* linear system  using backward substitution (Ch3 in the notes). For efficiency, we can use MATLAB’s built-in Gram-Shmidt process (since we're using floating point arithmetic) which simply requires a backslash operator on the rectangular matrix A – giving x (the diffusion tensor D).

Finally, to determine the magnitude of the diffusion at each voxel, we will find the mean diffusivity (mean of all three eigenvalues). To determine the fractional anisotrophy, (a measure of how the eigenvalues differ). Finally we must determine the principal diffusion direction, the eigenvector  which is associated with the largest eigenvalue , then use Matlab to produce an image with which it can be visualised.